the inner conductor can be truncated before the feed aperture, generating a TM_{01} mode in a circular waveguide. In both cases, the radiated field exhibits a null on the axis, and the required θ polarisation.

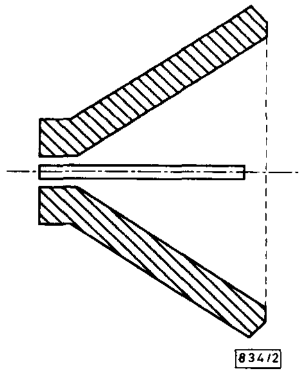

Fig. 2 *Schematic diagram of TEM feed*

Pattern analysis: The radiated field can be easily computed, if multiple interactions between reflectors are neglected, by first computing the primary radiation pattern, having assumed a known field on the feed aperture; if a TEM topography is taken, we obtain

$$E_f = \frac{\omega\mu}{R}\frac{V\cos\theta}{\ln a/b}\frac{S_0(u) - S_0(\frac{b}{a}u)}{u}e^{-jkR} \quad (1)$$

where a, b and V are, respectively, the outer, inner radius and voltage across the coaxial feed, $u = ka \sin\theta$, and

$$S_0(z) = 2\sum_{k=0}^{\infty} J_{2(1+k)}(z) \quad (2)$$

It is sufficient to take only three or four terms of this series to obtain accurate results in the range of interest.

It is therefore possible to compute via physical optics the induced surface current on the parabolic subreflector. The near field of the paraboloid is then computed, in correspondance with the surface of the conical main reflector, to obtain its currents: finally, the total far field is computed, adding to the main reflector scattered field the far field contribution of feed and subreflector.

Experimental results and discussion: An X-band prototype of the antenna has been constructed, to demonstrate the validity of the concept.

The subreflector is a 26cm parabolic dish, with 8cm focal distance; the conical main reflector diameter is slightly larger (30cm) to reduce the subreflector spillover, with additional flanges to give the structure more robustness. The subreflector is supported above the conical reflector by three thin dielectric rods, to minimise the scattering effects. The coaxial feed is tapered to an outer diameter of 30mm and is located in the tip area of the cone, truncating its upper part: it exhibits good matching in several frequency windows in the X and K bands.

Fig. 3 shows the computed and measured radiation pattern in the vertical plane of the antenna at 10.6GHz; the measurement angular range was limited to the main beam and first sidelobes for mechanical reasons.

The computed pattern accounts for the exact shape of the conical main reflector, including flanges, and for the shift of the feed phase centre due to the taper of the outer wall of the coaxial feed. Higher order effects, such as multiple interactions between the main reflector and the subreflector, are neglected. However, the comparison of these plots shows an excellent agreement in the main beam, and a couple of decibels difference in the sidelobes, probably due to the multiple interactions.

The theoretical analysis also shows that the spillover field from feed and subreflector has, in this configuration, some influence on the total vertical pattern. This is essentially due to the relatively small size of the main reflector (about 5 λ in height), so that its directivity is not very large with respect to those of feed and subreflector. However, the vertical 3dB beamwidth is ~10, very near to the theoretical value for a uniform aperture of the same height.

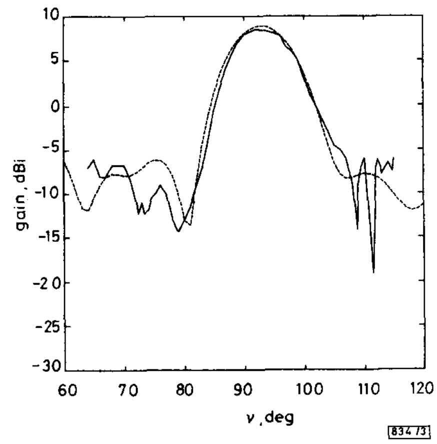

Fig. 3 *Theoretical and experimental vertical pattern of dual reflector antenna*

——— experimental
- - - - computed

This confirms the relatively high directivity of such a type of antenna.

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Asymptotic shaping gains from non-Gaussian distributions

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Indexing terms: Modems, Encoding

Optimum shaping gains are achieved when constellation symbols are sent in accordance with a Gaussian probability distribution. The Letter shows that the distribution can deviate from Gaussian and still give very good gains. It also gives asymptotic gains for two simple distributions.

Introduction: Constellation shaping: Constellation shaping is a technique included in the forthcoming V.34 modem standard which optimises the shape of the region defining a constellation in order to increase the power efficiency of the system [1]. We define a constellation C, whose size is $|C|$, as points from a translated lattice Λ lying in a region $\mathbb{R}$. In this Letter, Λ is the half-integer grid $Z^2+(1/2,1/2)$. The performance of this system can be measured by a coding gain based on the density of Λ and a shaping gain based on the shape of $\mathbb{R}$. Coding and shaping can be shown to be separable for large constellations [2]. That is, any gain achieved by shaping, the maximum is 1.53dB on white Gaussian noise channels, can be added to the gain already obtained by coding. After the initial 3 or 4dBs a small improvement in coding gain requires a sharp increase in complexity. Shaping appears to be ideal to provide some extra tenths of a decibel for little extra complexity.

By adequately choosing the limiting region $\mathbb{R}$, the average energy of the constellation C can be reduced for a given information rate R. The maximum gain of 1.53dB represents the difference between a constellation limited by a perfect N-dimensional sphere and a constellation limited by an N-dimensional cube (as $N \to \infty$). Limiting a constellation by an N-dimensional sphere is equivalent to selecting symbols from an infinite two-dimensional constellation using a Gaussian probability distribution. The size of the shaped constellation can be related to the constellation expansion ratio (CER) defined in the two-dimensional space as $|C|/2^R$ where 2^R is the size of the smallest uniform two-dimensional constellation able to transmit R bit/symbol. The shaping gain can be defined [3] as $\gamma_s = (2^R/6)/E_c$ where E_c is the average energy of the shaped constellation.

Infinite constellations have an infinite CER and are not practical. However, it can be shown [2] that truncating the probability distribution has little effect on the shaping gain for a particular CER range. For example, an ideally shaped 256 point QAM constellation for $R = 7$ bit/symbol, so $CER = 2$, gives a shaping gain of 1.52dB, only 0.01dB less than the maximum possible. Nearly all the maximum theoretical shaping gain can be achieved with finite CER. However, until now, no practical shaping scheme has approached this value for a reasonable complexity. In fact, none of the existing methods allows the modem designer to directly control the probability distribution and the entropy given by their shaping schemes for a given signal set. In particular none of the practical shaping schemes produces the ideal Gaussian probability distribution, but is this a problem? What sort of gain can simpler distributions give? Does a shaping method need to lead to a perfect Gaussian probability distribution to give extremely good shaping gains? This Letter answers these questions.

Non-Gaussian probability distributions: We impose different non-Gaussian probability distributions on two-dimensional circular constellations. Consider a family of distributions based on the uniform distribution f_1:

$$f_1(t) = \begin{cases} 1/T & \text{if } t \in [0, T] \\ 0 & \text{otherwise} \end{cases}$$

In fact we need to present our results for just three members of this family: f_2, f_3 and f_4, where

$$f_2(t) = f_1(t) * f_1(t) = \begin{cases} t/T^2 & \text{if } t \in [0, T] \\ (2T - t)/T^2 & \text{if } t \in [T, 2T] \\ 0 & \text{otherwise} \end{cases}$$

where * denotes convolution; with $f_3(t) = f_2(t) * f_1(t)$ and $f_4(t) = f_3(t) * f_1(t)$.

To measure how far from a Gaussian distribution the new distributions are, we use the index n. The central limit theorem tells us that convolving n uniform distributions ultimately leads to a Gaussian distribution. In this way the value of n leading to a reasonable shaping gain will give a good idea of the shape the distribution should have in order to perform well. These distributions are imposed on circular two-dimensional constellations with different CER values. For each case the parameter T is adjusted in order to give an entropy of 7 bit/symbol; we also check that the sum of all probabilities is equal to one. Table 1 presents the different shaping gains obtained under these conditions. For certain constellation expansions, the entropy cannot be 7: in that case we give the gain achieved for the minimum entropy possible (the value of the entropy is given in brackets).

From Table 1, it can be seen that simple distributions give very good gains. For low CER and as long as the entropy is optimum, the three distributions give neatly all of the possible shaping gain. Even f_2 which is a simple triangular distribution is only 0.049dB away from the optimum value for $CER = 1.281$. Therefore it does not seem too important to impose a perfect Gaussian probability distribution on finite two-dimensional constellations to obtain extremely good shaping gains. The second conclusion we can draw from these results is that the entropy is an important factor: as soon as the entropy increases, the gain falls quite quickly. However, if the difference in entropy is taken into account we find that the gain from each distribution is bounded by an upper limit. For instance, f_2 gives only 0.157dB gain for a CER of 1.687 and an entropy of 7.367. This difference of 0.367 bit/symbol is equivalent to an average energy of 1.105dB. Therefore if the entropy could be matched to the information rate, f_2 would give a shaping gain of 1.262dB which is approximately the last value of the gain achieved by the same distribution in optimal conditions (entropy = 7 bit/symbol, $CER = 1.281$ and $\gamma_s = 1.268$dB). Thus a shaping scheme leading to a probability distribution type f_2 and matching entropy and information rate would give very good gains but never more than 1.268dB. The same is true for f_3 where the limit is 1.452dB.

Table 1: Shaping gain for Gaussian and non-Gaussian distributions ($R = 7$)

$\|C\|$	CER	Shaping gain [dB]			
Distribution		f_2	f_3	f_4	Gaussian
140	1.094	1.008	1.010	1.010	1.010
148	1.156	1.154	1.160	1.160	1.160
156	1.219	1.230	1.252	1.253	1.253
164	1.281	1.268	1.316	1.317	1.317
172	1.344	1.159 (7.032)	1.358	1.359	1.361
180	1.406	1.009 (7.082)	1.389	1.393	1.395
188	1.469	0.858 (7.133)	1.413	1.419	1.422
192	1.5	0.708 (7.185)	1.422	1.429	1.433
208	1.625	0.569 (7.230)	1.447	1.462	1.468
216	1.687	0.157 (7.367)	1.451	1.472	1.489
232	1.812	-	1.452	1.484	1.504
240	1.875	-	1.413 (7.013)	1.489	1.51
248	1.937	-	1.299 (7.051)	1.492	1.515
256	2.0	-	1.187 (7.051)	1.495	1.52

Conclusion: Although a Gaussian probability distribution is optimum it is not needed to obtain very good shaping gains from practical QAM constellations. In particular, for small values of the CER, very simple distributions give nearly all of the achievable gain. Furthermore, we showed that the asymptotic gains produced by these simple distributions are bounded to values close to the optimum Gaussian gain when entropy loss is taken into account. Simple distributions can be obtained by convolving a uniform distribution: in practice this may be achieved by adding several random variables. A scheme using this idea to control the shape of the distribution will lead to very good gains if the entropy and information rate are matched.

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